A Online Appendix – Theories of Lawmaking

In this appendix, I provide the functional forms for the six benchmark theories. For the Majoritarian model, we have that,

$$x(s;\alpha) = \alpha_m \tag{1}$$

For the Filibuster Pivot Model, we have that,

$$x(s;\alpha) = \begin{cases} \alpha_m, & s \leq 2\alpha_l - \alpha_m \\ 2\alpha_l - s, & 2\alpha_l - \alpha_m \leq s \leq \alpha_l \\ s, & \alpha_l \leq s \leq \alpha_u \\ 2\alpha_u - s, & \alpha_u \leq s \leq 2\alpha_u - \alpha_m \\ \alpha_m, & s \geq 2\alpha_u - \alpha_m \end{cases} \tag{2}$$

For the remaining models, the policy outcome depends on the identity of the majority party. I let $x^D(s;\alpha)$ denote the policy outcome if the Democratic Party controls the chamber and I let $x^R(s;\alpha)$ denote the policy outcome if the Republican Party controls the chamber. I further assume that $\alpha_D < \alpha_m < \alpha_R$. For the Gatekeeping Model, we have that,

$$x^D(s;\alpha) = \begin{cases} \alpha_m, & s \leq 2\alpha_D - \alpha_m \\ s, & 2\alpha_D - \alpha_m \leq s \leq \alpha_m \\ \alpha_m, & s \geq \alpha_m \end{cases} \tag{3}$$

$$x^R(s;\alpha) = \begin{cases} \alpha_m, & s \leq \alpha_m \\ s, & \alpha_m \leq s \leq 2\alpha_R - \alpha_m \\ \alpha_m, & s \geq 2\alpha_R - \alpha_m \end{cases} \tag{4}$$

For the Setter model, we have that,

$$x^D(s;\alpha) = \begin{cases} \alpha_D, & s \leq \alpha_D \\ s, & \alpha_D \leq s \leq \alpha_m \\ 2\alpha_m - s, & \alpha_m \leq s \leq 2\alpha_m - \alpha_D \\ \alpha_D, & s \geq 2\alpha_m - \alpha_D \end{cases} \tag{5}$$

$$x^R(s;\alpha) = \begin{cases} \alpha_R, & s \leq 2\alpha_m - \alpha_R \\ 2\alpha_m - s, & 2\alpha_m - \alpha_R \leq s \leq \alpha_m \\ s, & \alpha_m \leq s \leq \alpha_R \\ \alpha_R, & s \geq \alpha_R \end{cases} \tag{6}$$

For the Gatekeeping-Filibuster Pivot model, we have that,

$$x^D(s;\alpha) = \begin{cases} \alpha_m, & s \leq 2\alpha_D - \alpha_m \\ s, & 2\alpha_D - \alpha_m \leq s \leq \alpha_u \\ 2\alpha_u - s, & \alpha_u \leq s \leq 2\alpha_u - \alpha_m \\ \alpha_m, & s \geq 2\alpha_u - \alpha_m \end{cases} \tag{7}$$

$$x^R(s;\alpha) = \begin{cases} \alpha_m, & s \leq 2\alpha_l - \alpha_m \\ 2\alpha_l - s, & 2\alpha_l - \alpha_m \leq s \leq \alpha_l \\ s, & \alpha_l \leq s \leq 2\alpha_R - \alpha_m \\ \alpha_m, & s \geq 2\alpha_R - \alpha_m \end{cases} \tag{8}$$

For the Setter-Filibuster Pivot model, we have that,

$$x^D(s;\alpha) = \begin{cases} \alpha_D, & s \leq \alpha_D \\ s, & \alpha_D \leq s \leq \alpha_u \\ 2\alpha_u - s, & \alpha_u \leq s \leq 2\alpha_u - \alpha_D \\ \alpha_D, & s \geq 2\alpha_u - \alpha_D \end{cases} \tag{9}$$

$$x^R(s;\alpha) = \begin{cases} \alpha_R, & s \leq 2\alpha_l - \alpha_R \\ 2\alpha_l - s, & 2\alpha_l - \alpha_R \leq s \leq \alpha_l \\ s, & \alpha_l \leq s \leq \alpha_R \\ \alpha_R, & s \geq \alpha_R \end{cases} \tag{10}$$

B Online Appendix – Evidence for the Proximity Model of Cosponsorship

In this appendix, I present some direct evidence that cosponsorship coalitions have a different form than voting coalitions. Specifically, I would like to distinguish between the proximity model (PM) and the relative proximity model (RPM) for cosponsorship. According to the proximity model, legislator n with ideal point α_n is more likely to cosponsor proposal t with location p_t if the proposal is close to the legislator's ideal point. If we assume that legislators have quadratic utility functions subject to an additive normally distributed stochastic shock, we have that the probability of cosponsorship is given by,

$$\Pr(y_{n,t} = 1) = \Phi(q_t - \rho(p_t - \alpha_n)^2) \tag{11}$$

Here, q_t is a proposal-specific parameter that captures the size of the cosponsorship coalition and $\rho > 0$ is the weight the legislator places on policy. According to the relative proximity model, the legislator is more likely to cosponsor the bill if he is closer to the proposal than to the status quo, s_t. We can conceptualize this as,

$$\Pr(y_{n,t} = 1) = \Phi(-\rho(p_t - \alpha_n)^2 + \rho(s_t - \alpha_n)^2) \tag{12}$$

We can encapsulate both (11) and (12) using the model,

$$\Pr(y_{n,t} = 1) = \Phi(a_t + b_t\alpha_n + c_t\alpha_n^2) \tag{13}$$

where $a_t = q_t - \rho p_t^2$, $b_t = -2\rho p_t$, and $c_t = -\rho$ for the proximity model and $a_t = -\rho(p_t^2 - s_t^2)$, $b_t = -2\rho(p_t - s_t)$, and $c_t = 0$ for the relative proximity model. Because $c_t = 0$ for the relative proximity model, $\Pr(y_{n,t} = 1)$ will be monotonic in α_n, indicating that end-against the middle coalitions are ruled out by the relative proximity model. By contrast, $\Pr(y_{n,t} = 1)$ will not be monotonic in α_n for the proximity model, indicating that ends against the middle coalitions are possible.

I consider three different ways of distinguishing between the proximity and relative proximity models for cosponsorship. These methods provide additional evidence in favor of the proximity model beyond the theoretical argument I provide in the text. In the body of the paper, I estimate ideal points using a combination of voting and cosponsorship data. Since my goal here is to test the proximity model of cosponsorship, I assume that the "true" legislator ideal points are well estimated by W-Nominate scores. The first approach estimates ideal points using cosponsorship data only according to both the proximity and relative proximity models and compares these results to conventional ideal point estimates. I find that there is a high correlation between ideal points estimated using the proximity model on cosponsorship data and conventional estimates. The correlation between ideal points estimated using the relative proximity model on cosponsorship data and conventional estimates is lower, suggesting that if cosponsorship and voting are governed by the same ideal points, the data are most consistent with the proximity model of cosponsorship. The second approach considers whether ends against the middle coalitions are present in cosponsorship data. I find that this is indeed the case, a fact that is inconsistent with the relative proximity of cosponsorship. The third approach estimates proximity and relative proximity models, taking the ideal points as fixed, and compares the fit of these two models. I find that the proximity model provides a superior fit.

B.1 Correlation between Cosponsorship and Voting Ideal Points

The first approach I employ estimates ideal points using cosponsorship data, according to the proximity and relative proximity models. The identification result I derive in Appendix C suggests that a very general proximity model of cosponsorship is identified if we also observe voting data for which the relative proximity model applies. This result cannot be applied here because we are interested in estimating ideal points based on cosponsorship data alone, and the model estimated in the main body of the paper is in fact not identified when only cosponsorship data is available. For this reason, here I estimate a different model,

$$\Pr(y_{n,t} = 1) = \Phi(-w_n - q_t - \theta' x_{n,t} - \rho(\alpha_n - p_t)^2) \tag{14}$$

This model of cosponsorship differs for the model I consider in the body of the paper because the variance of the error term is assume to be equal for all proposals. This model is more general than the most basic proximity model (see equation (11)) in that the cosponsorship threshold is allowed to vary over individuals. I have verified that the ideal points are globally identified in this model.[1]

In Table 1, I report the correlations and rank correlations between W-Nominate scores and cosponsorship ideal points estimated according to the proximity model, the relative proximity model, and the linear probability relative proximity model.
Cosponsorship ideal points estimated according to the proximity model are the most highly correlated with W-Nominate scores. If fact, the correlations are extremely high. The correlation is always higher for the proximity model than it is for the relative proximity model (including the linear probability variety advocated by Aleman et al. (2009)). The rank correlation is always higher for the proximity model than the two relative proximity models. The correlations for the relative proximity models, which initially seem reasonable, actually suggest poor performance because, with the exception of the 103rd congress, the correlation between W-Nominate scores and party is higher than the correlation between W-Nominate scores and either relative proximity estimates.

[1] A proof of the result is available from the author upon request.

Congress	Correl. w/ W-Nom.				Rank Correl. w/ W-Nom.		
	PM	*RPM*	*RPM-LP*	*Party*	*PM*	*RPM*	*RPM-LP*
103	0.93	0.92	0.92	0.91	0.94	0.92	0.93
104	0.96	0.93	0.92	0.95	0.95	0.92	0.92
105	0.97	0.93	0.95	0.95	0.95	0.93	0.93
106	0.96	0.94	0.94	0.96	0.96	0.94	0.94
107	0.96	0.89	0.88	0.94	0.94	0.88	0.88
108	0.94	0.89	0.86	0.96	0.95	0.91	0.90
109	0.96	0.93	0.92	0.95	0.95	0.93	0.92

Table 1: Alternative Cosponsorship Ideal Point Estimates for the U.S. Senate—This table reports the correlations and rank correlations between various cosponsorship ideal points estimates and W-Nominate scores, which are estimated using voting. Here, PM denotes the proximity model specified in (11), RPM denotes the (conventional) relative proximity model specified in (12), and RPM-LP denotes the Heckman-Snyder Linear Probability Model estimated using the principal components decomposition.

The Proximity Model, by contrast, achieves higher correlation than party in all congresses except the 108th. This evidence is most consistent with the theory that voting and cosponsorship are governed by the same ideal points, but that the proximity model best explains cosponsorship coalitions while the relative proximity model best explains voting coalitions.

My results actually help explain existing difficulties in estimating ideal points from cosponsorship data. Crisp, Desposato and Kanthak (2007) attempted to recover ideal points from cosponsorships data using the W-Nominate algorithm and found unsatisfactory results. They stressed that cosponsorship ideal point estimates where quite sensitive to the assumed data generating process. They suggested that individuals may fail to cosponsor because they are two far away from the status quo and that a two-cutpoint model may be necessary. The two-cutpoint model may improve the performance of the estimates because this model allows for ends against the middle coalitions. I argue that applying a proximity model accomplishes the same thing, and thus provides an improved model of cosponsorship.

B.2 Voting and Cosponsorship Coalitions

As I described earlier, the proximity model and the relative proximity model can nested in the following model,

$$\Pr(y_{n,t} = 1) = \Phi(a_t + b_t\alpha_n + c_t\alpha_n^2) \tag{15}$$

According to the relative proximity model, we must have $c_t = 0$. This implies that $\Pr(y_{n,t} = 1)$ is monotonic in α_n, indicating that we should not observe ends against the middle coalitions.

Voting coalitions on final passage votes can largely be described using a one-dimensional spatial model. Poole and Rosenthal (1997) considered whether estimating a two cutpoint model improved classification. They found that classification success was not much improved indicating that there are few ends against the middle voting coalitions. If the relative proximity model describes cosponsorship, than we should not observe ends against the middle cosponsorship coalitions. Alternatively, the proximity model allows for ends against the middle cosponsorship coalitions.

In Figure 1, I plot cosponsorship coalitions for the 103rd Senate. I report only bills for which

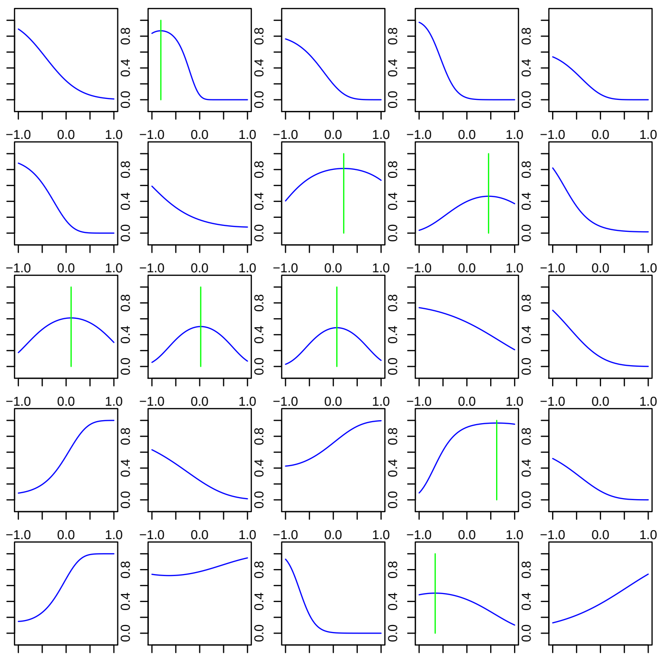

Figure 1: Probability of Cosponsoring (103rd Congress)—The blue line indicates predicted values based on (13). The green line indicates maximum predicted value when the maximum lies in the [-1,1] interval.

the number of cosponsors was greater than 25. I chose a high threshold of cosponsors to bias the results against finding many ends against the middle coalitions.[2] Among the 25 bills that meet this criteria, we see that 8 qualify as end-against-the-middle-coalitions. The presence of a large number of ends against the middle cosponsorship coalitions, even when the number of cosponsors is large, suggests that the relative proximity model is an inaccurate description of cosponsorship.

Some bills reach final passage without any intervening amendments. For such bills, if cosponsorship decisions are based on relative proximity, the parameters in equation (13) should be the same for the vote and the cosponsorship decisions. In general, only important pieces of legislation will see significant cosponsorship, and important pieces of legislation are typically successfully amended. In the 108th congress, we observe 5 bills with at least 5 cosponsors that received final passage votes on unamended bills. I plot the results for these bills in Figure 2. Two things are immediately apparent. First, even though the spatial locations of the proposal and status quo are the same for the cosponsorship and voting decisions, the cosponsorship coalitions are generally smaller. Second, in four of the cases, we observe small ends against the middle coalitions in the cosponsorship decision and large one-sided coalitions in the voting decision. These results are at odds with the relative proximity model of cosponsorship, but are consistent with the proximity model of cosponsorship.

[2]Any bill with only a handful of cosponsors is itself evidence against the relative proximity model unless the cosponsors are located at the edge of the policy space, but I would like to demonstrate that the relative proximity model fairs poorly even under conditions that are more favorable.

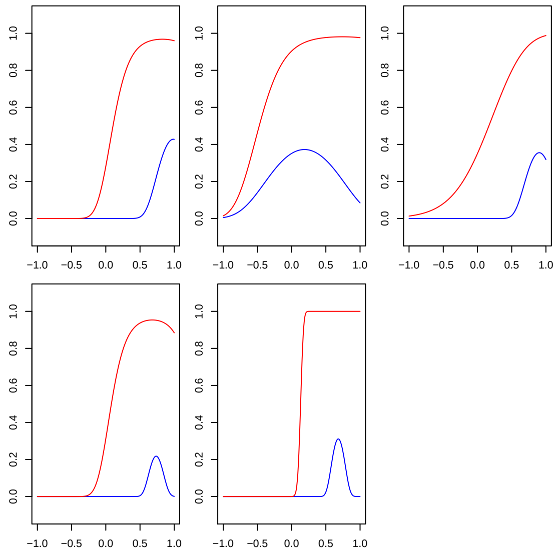

Figure 2: Probability of Cosponsoring and Voting Yea (108rd Congress)—The red line indicates predicted values based on (13) for voting coalitions and the blue line indicates predicted values for cosponsorship coalitions. The bills included here were not successfully amended, indicating that the proposal and status quo locations are identical for the cosponsorship and voting decisions.

B.3 Relative Model Fit

Here, I assess the relative fit of the proximity and relative proximity models for cosponsorship. I once against hold fixed the legislator ideal points at the W-Nominate estimates. I consider the following reduced form for the relative proximity model,

$$\Pr(y_{n,t} = 1) = \Phi(a_t + b_t \alpha_n) \tag{16}$$

and I consider the following form of the proximity model,

$$\Pr(y_{n,t} = 1) = \Phi(q_t - \rho(p_t - \alpha_n)^2) \tag{17}$$

I purposely depart from the more general proximity model presented in the paper—the proximity model I present in the paper allows for heteroskedastic error terms and allows the cosponsorship thresholds to vary with legislator characteristics. I consider a more restricted model to emphasize that the differences in model fit are not due to the fact that the proximity model has many more free parameters. I also note that the model I consider here is different than Poole and Rosenthal's two cutpoint model. Poole and Rosenthal's estimator has many more free parameters than a one-dimensional relative proximity model. The proximity model I estimate is more restrictive and has only one more parameter than the relative proximity model, indicating that the model fits can be compared without adjusting for the number of parameters.[3]

The model fit for the 103rd through 109th congresses is reported in Table 2. The results indicate that the proximity model consistently outperforms the relative proximity model. In all

[3]Poole and Rosenthal were investigating whether strategic voting leads to ends against the middle coalitions, in which case the more restrictive model I estimate would not be appropriate.

Congress	Log-Likelihood		Geo. Mean Prob.	
	RPM	*PM*	*RPM*	*RM*
103	-31142	-29515	0.925	0.933
104	-25944	-24973	0.928	0.936
105	-32751	-31338	0.922	0.931
106	-45499	-44224	0.915	0.923
107	-42368	-40566	0.918	0.926
108	-42612	-41961	0.912	0.919
109	-47040	-45869	0.929	0.935

Table 2: Model Fit for Cosponsorship Models using W-Nominate Ideal Points—This table reports the log-likelihood and geometric mean probability for the relative proximity and proximity models of cosponsorship when the ideal points are constrained to be equal to the W-Nominate scores.

seven congresses I consider, the proximity model achieves a higher log-likelihood and a higher geometric mean probability than the proximity model. The improvements in the geometric mean probability are small in magnitude because the baseline model fit for cosponsorship is high because most bills have few cosponsors.

B.4 Summary

In this appendix, I presented three pieces of evidence that cosponsorship decisions can be best explained by the proximity model. These results complement the theoretical argument and the results presented in the paper. They also suggest that the proximity model for cosponsorship provides a sound basis for the analysis performed here and in Woon (2008).

C Online Appendix – Identification

C.1 Model

In this appendix, I present the multi-dimensional version of the model and demonstrate that the parameters of the model are globally identified. I assume that there are N legislators who make T_v voting decisions and T_c cosponsorship decisions, where $T = T_v + T_c$. I let $\mathcal{T}_v$ denote the set of indices corresponding to voting decisions and I let $\mathcal{T}_c$ denote the set indices corresponding to cosponsorship decisions, where $\mathcal{T}_v \cap \mathcal{T}_c = \emptyset$, $\mathcal{T}_v \cup \mathcal{T}_c = \{1, 2, ..., T\}$, $T_v = |\mathcal{T}_v|$, and $T_c = |\mathcal{T}_c|$.

For each $t \in \mathcal{T}_v$, a proposal p_t is pitted against a status quo s_t. Legislator n receives utility $u^p_{n,t} = -(p_t - \alpha_n)'W(p_t - \alpha_n) + \varepsilon^p_{n,t}$ from voting yea on proposal t and receives utility $u^s_{n,t} = -(s_t - \alpha_n)'W(s_t - \alpha_n) + \varepsilon^s_{n,t}$ from voting nay on proposal t. Here, W is a symmetric positive definite weighting matrix that reflects the relative importance of different spatial dimensions.

Let $y_{n,t} = 1$ denote a yea vote and let $y_{n,t} = 0$ denote a nay vote for $t \in \mathcal{T}_v$. Assume that the legislator votes yea if and only if $u^p_{n,t} \geq u^s_{n,t}$. We have,

$$y_{n,t} = 1 \Leftrightarrow u^p_{n,t} \geq u^s_{n,t} \Leftrightarrow \varepsilon^p_{n,t} - \varepsilon^s_{n,t} \geq p'_t W p_t - s'_t W s_t - 2(p_t - s_t)' W \alpha_n \text{ for } t \in \mathcal{T}_v \quad (18)$$

Define $\varepsilon_{n,t} = \varepsilon^p_{n,t} - \varepsilon^s_{n,t}$. Suppose that $\varepsilon_{n,t}$ are independent across n and t and that $\varepsilon_{n,t}$ has cumulative distribution function $F(\varepsilon/\sigma_t)$, where F is strictly increasing. Here, I allow the variance of the error term to vary across votes (the importance of this assumption will become clear shortly). Under these assumptions, we obtain,[4]

$$\Pr(y_{n,t} = 1; \alpha, p, s, \sigma, W) = F\left(\frac{p_t'Wp_t - s_t'Ws_t - 2(p_t - s_t)'W\alpha_n}{\sigma_t}\right) \text{ for } t \in \mathcal{T}_v \tag{19}$$

A common reparameterization is to set,

$$\gamma_t = \frac{p_t'Wp_t - s_t'Ws_t}{\sigma_t} \text{ for } t \in \mathcal{T}_v \tag{20}$$

$$\beta_t = \frac{-2W(p_t - s_t)}{\sigma_t} \text{ for } t \in \mathcal{T}_v \tag{21}$$

in which case we have, $\Pr(y_{n,t} = 1; \alpha, \gamma, \beta) = F(\gamma_t + \beta_t'\alpha_n)$. This reparameterization is useful if the goal is to recover only α, but hinders our ability to recover (p, s), which are of direct interest in our case.

For each $t \in \mathcal{T}_c$, each legislator faces the choice of whether to cosponsor proposal p_t. I assume that legislator n will choose to cosponsor proposal t if the utility $u_{n,t}^c$ is greater than some threshold, $\bar{u}_{n,t}$. I assume that the utility function is quadratic, $u_{n,t}^c = -(p_t - \alpha_n)'W(p_t - \alpha_n)$. I assume that the threshold is random, but that the mean varies by legislator and by proposal. Specifically, I assume that $\bar{u}_{n,t} = \theta'x_n + q_t + \varepsilon_{n,t}^c$ where $\varepsilon_{n,t}^c$ has cdf $F(\varepsilon/\delta_t)$. Here, x_n is a vector of legislator-specific covariates and q_t is a bill-specific fixed effect. I also allow for a heteroskedastic error term in the cosponsorship threshold.[5] We have,

$$y_{n,t} = 1 \Leftrightarrow u_{n,t}^c \geq \bar{u}_{n,t} \Leftrightarrow -(p_t - \alpha_n)'W(p_t - \alpha_n) - \theta'x_n - q_t - \geq \varepsilon_{n,t}^c \text{ for } t \in \mathcal{T}_c \tag{22}$$

This yields,

$$\Pr(y_{n,t} = 1 | x_n; \alpha, p, q, \delta, W, \theta) = F\left(\frac{-(p_t - \alpha_n)'W(p_t - \alpha_n) - \theta'x_n - q_t}{\delta_t}\right) \text{ for } t \in \mathcal{T}_c \tag{23}$$

Combining this with the previous result, we have the following statistical model,

$$\Pr(y_{n,t} = 1 | x_n; \alpha, p, s, q, \sigma, \delta, W, \theta) = \begin{cases} F\left(\frac{s_t'Ws_t - p_t'Wp_t - 2(p_t - s_t)'W\alpha_n}{\sigma_t}\right), & t \in \mathcal{T}_v \\ F\left(\frac{-(p_t - \alpha_n)'W(p_t - \alpha_n) - \theta'x_n - q_t}{\delta_t}\right), & t \in \mathcal{T}_c \end{cases} \tag{24}$$

My goals in this section are as follows. First, I would like to demonstrate that the location of the proposal and the status quo cannot be recovered from voting data alone, *under reasonable assumptions*. Second, I would like to demonstrate the location of the proposal can be recovered using a combination of voting data and cosponsorship data, under a similar set of assumptions. Third, I would like to demonstrate that the status quo can be recovered if we know the location

[4]Throughout, I use a letter without a subscript to denote a vector of parameters, i.e. $\alpha = (\alpha_1, ..., \alpha_N)$ and $\sigma = (\sigma_1, ..., \sigma_{T_v})$.

[5]As I later claim that the potential for heteroskedasticity in the error term is one of the reasons we cannot identify proposal and status quo locations from voting data alone, I must allow for heteroskedasticity here.

of the proposal and the cutting line. To demonstrate each of these claims, I will study global identification of the parameters of interest.[6]

C.2 Identification

Let $y_{n,t}$ be the data, let ω denote the model parameters, and let $\Pr(y_{n,t} = 1; \omega)$ for $n \in \mathcal{N}$ and $t \in \mathcal{T}$ denote the statistical model. Partition the parameters $\omega = (\kappa, \eta)$ into the parameters of interest κ and the nuisance parameters η. Let K denote the space of parameters of interest and let N denote the space of nuisance parameters. Throughout, I use zero subscripts to denote the parameters of the data generating process. That is, $\omega_0 = (\kappa_0, \eta_0)$ are the "true" parameter values.

Definition 1. *κ_0 is identified if there does not exist a $(\kappa, \eta) \in \mathrm{K} \times \mathrm{N}$ with $\kappa \neq \kappa_0$ such that $\Pr(y_{n,t} = 1; \kappa, \eta) = \Pr(y_{n,t} = 1; \kappa_0, \eta_0)$ for all $n \in \mathcal{N}$ and $t \in \mathcal{T}$.*

Essentially, the definition states that the parameter of interest is identified if there does not exists an alternative value for the parameter of interest in the parameter space that would lead to the same distribution for the data as the true parameter of interest. The definition is relevant because such a condition is always required to demonstrate the consistency of an estimator. If this condition were to fail, it would suggest that the likelihood function does not vary with κ. This, in turn, means that we cannot obtain a consistent point estimator of κ_0. Notice, however, that we only require identification of the parameter of interest. This is important in our application because σ and δ are not theoretically relevant (or at least less theoretically relevant).

C.3 Identification from Voting Data

The goal in standard ideal point estimation is to recover α_0, where $(p_0, s_0, \sigma_0, W_0)$ are considered nuisance parameters. If we are interested in testing theories of lawmaking, (p_0, s_0) are no longer nuisance parameters. Instead, the parameters of interest are (α_0, p_0, s_0) while (σ_0, W_0) are nuisance parameters. I first consider identification of (α_0, p_0, s_0) when $T_c = 0$. That is, I consider identification of the ideal points, proposal locations, and status quo locations using voting data alone.

Applying Definition 1, (α_0, p_0, s_0) are identified if there does not exist an $(\alpha, p, s, \sigma, W)$ with $(\alpha, p, s) \neq (\alpha_0, p_0, s_0)$ such that,

$$F\left(\frac{p_t' W p_t - s_t' W s_t - 2(p_t - s_t)' W \alpha_n}{\sigma_t}\right) =$$

$$F\left(\frac{p_{t,0}' W_0 p_{t,0} - s_{t,0}' W_0 s_{t,0} - 2(p_{t,0} - s_{t,0})' W_0 \alpha_{n,0}}{\sigma_{t,0}}\right) \text{ for all } n \in \mathcal{N} \text{ and } t \in \mathcal{T} \tag{25}$$

The policy space is normalized by fixing the first $D+1$ ideal points, where the vectors $\{\alpha_d - \alpha_{D+1}\}_{d=1}^{D}$ span $\mathbb{R}^D$. Without loss of generality, I choose $\alpha_{n,d} = \alpha_{n,d,0} = e_d$ for $d \in \{1, ..., D\}$ and $\alpha_{n,D+1} = \alpha_{n,D+1,0} = 0$, where e_d is the d^{th} unit vector. Hence, we have,

[6] Rivers (2003) studies local identification of voter ideal points using roll call data. A direct proof of global identification is actually more straightforward than the indirect proof techniques for local identification used by Rivers, as long as we are willing to commit to a particular normalization beforehand. In addition, I note that identification in the presence of nuisance parameters cannot easily be studied using River's proof technique.

$$\mathrm{K} = \{(\alpha, p, s) \in \mathbb{R}^{N+2T} : \alpha_1 = e_1, ..., \alpha_D = e_D, \alpha_{D+1} = 0\} \tag{26}$$

$$\mathrm{N} = \{(\sigma, W) \in (\mathbb{R}^{++})^T : \sigma_1 = 1, W \text{ s.p.d.}\} \tag{27}$$

The model is not identified because any $(\alpha, p, s, \sigma, W) \in \mathrm{K} \times \mathrm{N}$ satisfying,

$$\alpha_n = \alpha_{n,0} \text{ for } n \in \mathcal{N} \tag{28}$$

$$p_t = \tfrac{1}{2}(I + \tfrac{\sigma_t}{\sigma_{t,0}} W^{-1} W_0) p_{t,0} + \tfrac{1}{2}(I - \tfrac{\sigma_t}{\sigma_{t,0}} W^{-1} W_0) s_{t,0} \text{ for } t \in \mathcal{T} \tag{29}$$

$$s_t = \tfrac{1}{2}(I - \tfrac{\sigma_t}{\sigma_{t,0}} W^{-1} W_0) p_{t,0} + \tfrac{1}{2}(I + \tfrac{\sigma_t}{\sigma_{t,0}} W^{-1} W_0) s_{t,0} \text{ for } t \in \mathcal{T} \tag{30}$$

will satisfy (25).[7] Identification fails for three reasons. First, we cannot distinguish between a high degree of dispersion (the difference between the proposal and the status quo) and a noisy vote. Using (29) and (30), we have,

$$p_t - s_t = \tfrac{\sigma_t}{\sigma_{t,0}} W^{-1} W_0 (p_{t,0} - s_{t,0}) \tag{31}$$

If we increase the dispersion $p_t - s_t$, we can compensate by increasing the relative error variance $\frac{\sigma_t}{\sigma_{t,0}}$ as well. There is no reason to assume, a priori, that the error variances are homoskedastic. A particular vote may be noisy, for example, because it contains a high degree of particularistic content.

Even if we assume that the error variances are homoskedastic (i.e. $\sigma_t = \sigma_{t,0} = 1$ for $t \in \mathcal{T}$), problems remain. The second problem is that we cannot distinguish between high aggregate dispersion and a large weight placed on policy utility. Plugging in $\sigma_t = \sigma_{t,0} = 1$ into (31), we have,

$$p_t - s_t = W^{-1} W_0 (p_{t,0} - s_{t,0}) \tag{32}$$

If we increase the dispersion of any component of $p_t - s_t$ by the same amount for each $t \in \mathcal{T}$, we can compensate by adjusting W accordingly (even in the one-dimensional case).

Finally, even if we assume that the utility function satisfies $W = W_0 = I$, we have that any (α, p, s) satisfying,

$$\alpha_n = \alpha_{n,0} \text{ for } n \in \mathcal{N} \tag{33}$$

$$\begin{bmatrix} I & -I \\ p_{t,0} - s_{t,0} & 0 \end{bmatrix} \begin{bmatrix} p_t \\ s_t \end{bmatrix} = \begin{bmatrix} p_{t,0} - s_{t,0} \\ (p_{t,0} - s_{t,0})' p_{t,0} \end{bmatrix} \text{ for } t \in \mathcal{T} \tag{34}$$

will satisfy (25). For each t, $(p_t, s_t) = (p_{t,0}, s_{t,0})$ clearly solve equation (34), but there will be multiple solutions provided $D > 1$ since,

$$rank\left(\begin{bmatrix} I & -I \\ p_{t,0} - s_{t,0} & 0 \end{bmatrix}\right) \leq D + 1 \tag{35}$$

[7] These are not the only such solutions, but this suffices to show that identification fails.

This is not to say that the restriction of $W = W_0 = I$ should be considered reasonable. If we make such an assumption, we are implicitly normalizing $D(D+1)/2$ coordinates of the policy space, in which case we are no longer free to normalize the ideal points of $D+1$ legislators.

The first and third problem are discussed in Poole (2005) while the second problem has not been previously considered. I note that the existing approaches of recovering ideal points from voting data remain valid. For example, Clinton, Jackman and Rivers (2004) derive a reduced form item response model for the case where the error variances are homoskedastic and $W = W_0 = I$. Poole (2005) allows the error variances to be heteroskedastic, but requires $W = W_0 = I$. In both cases, the restrictive models lead to the same reduced form as my framework, as long as (p_0, s_0) are not of direct interest to the researcher. When (p_0, s_0) are parameters of interest, these derivations can be misleading because the various restrictions employed are no longer without loss of generality.

C.4 Identification of Proposal Locations

I next consider identification using both cosponsorship and voting data.[8] The result below establishes the main result of the paper. Identification requires a number of normalizations as well a few substantive conditions on the data generating process which are likely to hold.

Proposition 1. *Suppose that $\alpha_{n,0} = e_n$ for $n \in \{1, ..., D\}$, $\alpha_{D+1,0} = 0$, and $\delta_{1,0} = 1$. Suppose that the vectors $\{p_{t,0} - s_{t,0}\}_{t \in \mathcal{T}_v}$ span $\mathbb{R}^D$. Suppose that W_0 is a symmetric positive definite matrix. Define the matrix A by,*

$$A_{\frac{i(i-1)}{2}+j,n} = \begin{cases} \alpha_{n,i,0}\alpha_{n,j,0} - \alpha_{n,i,0}, & i = j \\ \alpha_{n,i,0}\alpha_{n,j,0}, & i \neq j \end{cases} \quad \text{for } j \leq i, (i,j) \in \{1, ..., D\} \text{ and } n \in \mathcal{N} \tag{36}$$

$$A_{\frac{D(D-1)}{2}+k,n} = x_{n,k} - x_{D+1,k} - \sum_{d=1}^{D} (x_{d,k} - x_{D+1,k})\alpha_{n,d,0} \text{ for } k \in 1, ..., K \text{ and } n \in \mathcal{N} \tag{37}$$

Suppose that,

$$rank(A) \geq \tfrac{D(D+1)}{2} + K \tag{38}$$

$$diag\{W_0\}'\alpha_{n,0} - \alpha_{n,0}' W_0 \alpha_{n,0} \neq 0 \text{ for some } n \in \mathcal{N} \tag{39}$$

There does not exist a $(\alpha, \gamma, \beta, q, p_C, \delta, \theta, W) \neq (\alpha_0, \gamma_0, \beta_0, q_0, p_{C,0}, \delta_0, \theta_0, W_0)$ with $\alpha_n = e_n$ for $n \in \{1, ..., D\}$, $\alpha_{D+1} = 0$, $\gamma_t = \frac{p_t' W p_t - s_t' W s_t}{\sigma_t}$, $\beta_t = \frac{-2W(p_t - s_t)}{\sigma_t}$, and $\delta_1 = 1$ such that,[9]

$$F\left(\frac{s_t' W s_t - p_t' W p_t - 2(p_t - s_t)' W \alpha_n}{\sigma_t}\right)$$

[8] I have verified that $(\alpha, q, p, \delta, W, \theta)$ cannot be identified from cosponsorship data alone, but I do not report the result here.

[9] Define p_C to be the vector of p_t with $t \in \mathcal{T}_C$ and $p_{C,0}$ to be the vector of $p_{t,0}$ with $t \in \mathcal{T}_C$.

$$= F\left(\frac{s_{t,0}'W_0 s_{t,0} - p_{t,0}'W_0 p_{t,0} - 2(p_{t,0} - s_{t,0})'W_0\alpha_{n,0}}{\sigma_{t,0}}\right) \textit{ for all } n \in \mathcal{N} \textit{ and } t \in \mathcal{T}_v \tag{40}$$

$$F\left(\frac{-(p_t - \alpha_n)'W(p_t - \alpha_n) - \theta' x_n - q_t}{\delta_t}\right)$$

$$= F\left(\frac{-(p_{t,0} - \alpha_{n,0})'W_0(p_{t,0} - \alpha_{n,0}) - \theta_0' x_n - q_{t,0}}{\delta_{t,0}}\right) \textit{ for all } n \in \mathcal{N} \textit{ and } t \in \mathcal{T}_c \tag{41}$$

The proposition imposes a number of normalizations on the parameters space. The first $D+1$ ideal points are normalized and the variance of the cosponsorship error for proposal 1 is normalized to one. The difference between the proposals and status quos is assumed to span $\mathbb{R}^D$, a condition which is standard (Rivers, 2003). The weighting matrix is assumed to be symmetric and positive definite, which further implies that the weighting matrix is invertible. None of these conditions are substantive. The two substantive conditions are given in equations (38) and (39). The second condition is clearly very weak—in the one dimensional case, it will hold if there is at least one individual with a true ideal point that does not equal zero or one. The first condition requires that the matrix A, whose value depends on the data generating process parameters α_0 and x, has sufficiently large rank. The matrix A has N rows, so it is quite likely that the rank of A will be greater than $\frac{D(D+1)}{2} + K$ as long as D and K are not too large. If $N > \frac{D(D+1)}{2} + K$, it will hold generically over $\alpha_0 \in \mathbb{R}^{ND}$ for a given x.

Proof of Proposition 1. I first show that $(\alpha_0, \gamma_0, \beta_0)$ is identified using voting data. First, since F is strictly increasing, (40) is equivalent to,

$$\frac{s_t'W s_t - p_t'W p_t - 2(p_t - s_t)'W\alpha_n}{\sigma_t}$$

$$= \frac{s_{t,0}'W_0 s_{t,0} - p_{t,0}'W_0 p_{t,0} - 2(p_{t,0} - s_{t,0})'W_0\alpha_{n,0}}{\sigma_{t,0}} \text{ for } n \in \mathcal{N} \text{ and } t \in \mathcal{T}_v \tag{42}$$

Using the fact that $\alpha_{D+1} = \alpha_{D+1,0} = 0$, we obtain,

$$\frac{s_t'W s_t - p_t'W p_t}{\sigma_t} = \frac{s_{t,0}'W_0 s_{t,0} - p_{t,0}'W_0 p_{t,0}}{\sigma_{t,0}} \text{ for } t \in \mathcal{T}_v \tag{43}$$

Using (20) and (66), we have that,

$$\gamma_t = \gamma_{t,0} \text{ for } t \in \mathcal{T}_v \tag{44}$$

Next, subtracting (43) from (42), we obtain,

$$\frac{-2(p_t - s_t)'W\alpha_n}{\sigma_t} = \frac{-2(p_{t,0} - s_{t,0})'W_0\alpha_{n,0}}{\sigma_{t,0}} \text{ for } n \in \mathcal{N} \text{ and } t \in \mathcal{T}_v \tag{45}$$

Using the fact that $\alpha_d = \alpha_{d,0} = e_d$ for $d \in \{1, ..., D\}$, we have,

$$\frac{-2(p_t - s_t)'We_d}{\sigma_t} = \frac{-2(p_{t,0} - s_{t,0})'W_0 e_d}{\sigma_{t,0}} \text{ for } n \in \mathcal{N},\ t \in \mathcal{T}_v, \text{ and } d \in \{1, ..., D\} \tag{46}$$

This implies that,

$$\frac{(p_t - s_t)'W}{\sigma_t} = \frac{(p_{t,0} - s_{t,0})'W_0}{\sigma_{t,0}} \text{ for } n \in \mathcal{N} \text{ and } t \in \mathcal{T}_v \tag{47}$$

Using (21) and (67), we have that,

$$\beta_t = \beta_{t,0} \text{ for } t \in \mathcal{T}_v \tag{48}$$

Next, plugging (47) into (45), we obtain,

$$(p_{t,0} - s_{t,0})'W_0(\alpha_n - \alpha_{n,0}) = 0 \text{ for } n \in \mathcal{N} \text{ and } t \in \mathcal{T}_v \tag{49}$$

The vectors $\{p_{t,0} - s_{t,0}\}_{t \in \mathcal{T}_v}$ span $\mathbb{R}^D$, so it follows that,

$$\alpha_n = \alpha_{n,0} \text{ for } n \in \mathcal{N} \tag{50}$$

Hence, we have that $(\alpha_0, \gamma_0, \beta_0)$ is identified from the cosponsorship data alone.

Next, consider the cosponsorship data. Notice that (41) equivalent to,

$$\delta_{t,0}q_t + \delta_{t,0}\theta' x_n + \delta_{t,0}\alpha'_{n,0}W\alpha_{n,0} + \delta_{t,0}p'_t W p_t - 2\delta_{t,0}p'_t W\alpha_{n,0} = \delta_t q_{t,0} + \delta_t\theta'_0 x_n$$

$$+\delta_t\alpha'_{n,0}W_0\alpha_{n,0} + \delta_t p'_{t,0}W_0 p_{t,0} - 2\delta_t p'_{t,0}W_0\alpha_{n,0} \text{ for } n \in \mathcal{N} \text{ and } t \in \mathcal{T}_c \tag{51}$$

Using the fact that $\alpha_{D+1} = \alpha_{D+1,0} = 0$, we obtain,

$$\delta_{t,0}q_t + \delta_{t,0}\theta' x_{D+1} + \delta_{t,0}p'_t W p_t = \delta_t q_{t,0} + \delta_t\theta'_0 x_{D+1} + \delta_t p'_{t,0}W_0 p_{t,0} \text{ for } t \in \mathcal{T}_c \tag{52}$$

Subtracting (52) from (51), we obtain,

$$\delta_{t,0}\theta'(x_n - x_{D+1}) + \delta_{t,0}\alpha'_{n,0}W\alpha_{n,0} - 2\delta_{t,0}p'_t W\alpha_{n,0} = \delta_t\theta'_0(x_n - x_{D+1})$$

$$+\delta_t\alpha'_{n,0}W_0\alpha_{n,0} - 2\delta_t p'_{t,0}W_0\alpha_{n,0} \text{ for } n \in \mathcal{N} \text{ and } t \in \mathcal{T}_c \tag{53}$$

Using the fact that $\alpha_d = \alpha_{d,0} = e_d$ for $d \in \{1, ..., D\}$, we have,

$$\delta_{t,0}\theta'(x_d - x_{D+1}) + \delta_{t,0}[W]_{d,d} - 2\delta_{t,0}p'_t W e_d$$

$$= \delta_t\theta'_0(x_d - x_{D+1}) + \delta_t[W_0]_{d,d} - 2\delta_t p'_{t,0}W_0 e_d \text{ for } d \in \{1, ..., D\} \text{ and } t \in \mathcal{T}_c \tag{54}$$

Let $\Delta x = [x_1 - x_{D+1}; ...; x_D - x_{D+1}]$. Stacking these by column,

$$Wp_t = \tfrac{1}{2}(\theta - \tfrac{\delta_t}{\delta_{t,0}}\theta_0)'\Delta x + \tfrac{1}{2}diag\{W\} - \tfrac{1}{2}\tfrac{\delta_t}{\delta_{t,0}}diag\{W_0\} + \tfrac{\delta_t}{\delta_{t,0}}W_0 p_{t,0} \text{ for } t \in \mathcal{T}_c \tag{55}$$

Plugging (55) into (53),

$$\delta_{t,0}\alpha'_{n,0}W\alpha_{n,0} - \delta_t\alpha'_{n,0}W_0\alpha_{n,0} + \delta_t diag\{W_0\}'\alpha_{n,0} - \delta_{t,0}diag\{W\}'\alpha_{n,0}$$

$$+(\delta_{t,0}\theta - \delta_t\theta_0)'(x_n - x_{D+1}) - (\delta_{t,0}\theta - \delta_t\theta_0)'\Delta x'\alpha_{n,0} = 0 \text{ for } n \in \mathcal{N} \text{ and } t \in \mathcal{T}_c \tag{56}$$

Using $\delta_1 = \delta_{1,0} = 1$, we have,

$$\alpha'_{n,0}(W - W_0)\alpha_{n,0} - (diag\{W\} - diag\{W_0\})'\alpha_{n,0}$$

$$+(\theta - \theta_0)'[(x_n - x_{D+1}) - \Delta x\alpha_{n,0}] = 0 \text{ for all } n \in \mathcal{N} \tag{57}$$

This system is equivalent to,[10]

$$A'(vech(W - W_0), \theta - \theta_0) = 0 \tag{58}$$

By assumption, $rank(A) \geq \frac{D(D+1)}{2} + K$, implying that the unique solution to (58) is,

$$(W, \theta) = (W_0, \theta_0) \tag{59}$$

We can plug (59) into (56) to obtain,

$$(\delta_{t,0} - \delta_t)\left\{\alpha'_{n,0}W_0\alpha_{n,0} + diag\{W_0\}'\alpha_{n,0} + \theta'_0(x_n - x_{D+1}) - \theta'_0\Delta x\alpha_{n,0}\right\} = 0$$

$$\text{for } n \in \mathcal{N} \text{ and } t \in \mathcal{T}_c \tag{60}$$

By assumption,

$$\alpha'_{n,0}W_0\alpha_{n,0} + diag\{W_0\}'\alpha_{n,0} + \theta'_0(x_n - x_{D+1}) - \theta'_0\Delta x\alpha_{n,0} \neq 0 \text{ for some } n \tag{61}$$

so we have,

$$\delta_t = \delta_{t,0} \text{ for } t \in \mathcal{T}_c \tag{62}$$

We plug (59) and (62) into (55) to obtain,

$$p_t = p_{t,0} \text{ for } t \in \mathcal{T}_c \tag{63}$$

We plug (59), (62), and (63) into (52) to obtain,

$$q_t = q_{t,0} \text{ for } t \in \mathcal{T}_c \tag{64}$$

proving the result. □

C.5 Identification of Status Quo Locations

A final step is to recover the location of the final form of the bill and the status quo. I let $l(t) \in \mathcal{T}_c$ denote the index of the last proposal that passed for the original bill associated with final passage vote t. I let $t \in \mathcal{T}_f$ index final passage votes, where $\mathcal{T}_f \subset \mathcal{T}_v$. I assume that $l(t)$ is known and that for each $t \in \mathcal{T}_f$,

[10] For a n by n matrix, we define $vech(A)$ by $vech(A)_{\frac{i(i-1)}{2}+j} = A_{i,j}$ where $j \leq i$ and $i, j \in \{1, ..., n\}$.

$$p_{l(t),0} = p_{t,0} \text{ for all } t \in \mathcal{T}_f \tag{65}$$

This amounts to assuming that for each final passage vote, we can identify the final proposal that passed and that we observe cosponsorship behavior for this proposal.

We can identify the status quo if we observe the results of a final passage vote. By the definition of the reduced form parameters, we have,

$$\gamma_{t,0} = \frac{p'_{t,0} W_0 p_{t,0} - s'_{t,0} W_0 s_{t,0}}{\sigma_{t,0}} \text{ for } t \in \mathcal{T}_v \tag{66}$$

$$\beta_{t,0} = \frac{-2W_0(p_{t,0} - s_{t,0})}{\sigma_{t,0}} \text{ for } t \in \mathcal{T}_v \tag{67}$$

From Proposition 1, it follows that W_0 is identified, $(\gamma_{t,0}, \beta_{t,0})$ are identified for all $t \in \mathcal{T}_f$, and $p_{l(t),0}$ are identified for all $l \in \mathcal{T}_c$. I will show that equations (65), (66), and (67) imply that $(p_{t,0}, s_{t,0}, \sigma_{t,0})$ are identified for all $t \in \mathcal{T}_f$. For each $t \in T_f$, equations (65), (66), and (67) define a system of $2D+1$ with $2D+1$ unknowns (the unknowns are p_t, s_t, and σ_t). These equations are nonlinear, so there is no a priori guarantee that there is a unique solution.

The following proposition shows that the solution is, in fact, unique.

Proposition 2. *Suppose that for* $t \in \mathcal{T}_f$, $\beta_{t,0} \neq 0$ *and* $\sigma_{t,0} \neq 0$. *If* $p_{l(t),0} = p_{t,0}$, $\gamma_{t,0} = \frac{p'_{t,0} W_0 p_{t,0} - s'_{t,0} W_0 s_{t,0}}{\sigma_{t,0}}$, *and* $\beta_{t,0} = \frac{-2W_0(p_{t,0} - s_{t,0})}{\sigma_{t,0}}$, *then the following must hold,*

$$s_{t,0} = p_{l(t),0} - 2\left[\frac{\gamma_{t,0} + p'_{l(t),0}\beta_{t,0}}{\beta'_{t,0} W_0^{-1} \beta_{t,0}}\right] W_0^{-1} \beta_{t,0} \tag{68}$$

$$\sigma_{t,0} = \frac{-2\beta'_{t,0}(p_{l(t),0} - s_{t,0})}{\beta'_{t,0} W_0^{-1} \beta_{t,0}} \tag{69}$$

The conditions for identification here are once again quite weak. In the one-dimensional case, we simply require that $\beta_{t,0}$ is not equal to zero, or that the vote discriminates on ideology. This will hold provided that $p_{t,0} \neq s_{t,0}$, or provided that the proposal and status quo location are not identical. In the unlikely event that this is the case, we will be able to determine this fact, but we will not be able to identify $\sigma_{t,0}$. When $D = 1$, we can show that (68) reduces to $\frac{1}{2}(s_{t,0} + p_{l(t),0}) = -\frac{\gamma_{t,0}}{\beta_{t,0}}$, which is equivalent to the conventional formula relating the proposal, the status quo, and the cutpoint in the one-dimensional case (i.e. the cutpoint is the average of the proposal and the status quo).

Proof of Proposition 2. Plugging in $p_{l(t),0} = p_{t,0}$ into the other equations, we have,

$$\gamma_{t,0} = \frac{p'_{l(t),0} W_0 p_{l(t),0} - s'_{t,0} W_0 s_{t,0}}{\sigma_{t,0}} \tag{70}$$

$$\beta_{t,0} = \frac{-2W_0(p_{l(t),0} - s_{t,0})}{\sigma_{t,0}} \tag{71}$$

If $\gamma_{t,0} = 0$, then (71) implies that,

$$0 = \frac{p'_{l(t),0}W_0 p_{l(t),0} - s'_{t,0}W_0 s_{t,0}}{\sigma_{t,0}} = \frac{(p_{l(t),0} + s_{t,0})' W_0 (p_{l(t),0} - s_{t,0})}{\sigma_{t,0}} \tag{72}$$

Since $\beta_{t,0} \neq 0$, we must have $s_{t,0} = -p_{l(t),0}$, which implies (68). Alternatively, if $\gamma_{t,0} \neq 0$, we can divide (71) by (70) to obtain,

$$\frac{\beta_{t,0}}{\gamma_{t,0}} = \frac{-2W_0(p_{l(t),0} - s_{t,0})}{p'_{l(t),0}W_0 p_{l(t),0} - s'_{t,0}W_0 s_{t,0}} \tag{73}$$

We can rearrange this to obtain,

$$\frac{p'_{l(t),0}W_0 p_{l(t),0} - s'_{t,0}W_0 s_{t,0}}{-2\gamma_{t,0}} {W_0}^{-1}\beta_{t,0} = p_{l(t),0} - s_{t,0} \tag{74}$$

or equivalently,[11]

$$\frac{p'_{l(t),0}W_0 p_{l(t),0} - s'_{t,0}W_0 s_{t,0}}{-2\gamma_{t,0}} = \frac{p_{l(t),d,0} - s_{t,d,0}}{[{W_0}^{-1}\beta_{t,0}]_d} \text{ for } d \in \{1, ..., D\} \tag{75}$$

It follows that,

$$\frac{p_{l(t),d,0} - s_{t,d,0}}{[{W_0}^{-1}\beta_{t,0}]_d} = \frac{p_{t,1,0} - s_{t,1,0}}{[{W_0}^{-1}\beta_{t,0}]_1} \text{ for } d \in \{1, ..., D\} \tag{76}$$

which implies that,

$$s_{t,0} = p_{l(t),0} - \frac{p_{l(t),1,0} - s_{t,1,0}}{[{W_0}^{-1}\beta_{t,0}]_1} {W_0}^{-1}\beta_{t,0} \tag{77}$$

Plugging (77) into (74), we obtain,

$$\left[\beta'_{t,0}p_{l(t),0} + \gamma_{t,0} - \tfrac{1}{2}\frac{p_{l(t),1,0} - s_{t,1,0}}{[{W_0}^{-1}\beta_{t,0}]_1}\beta'_{t,0}{W_0}^{-1}\beta_{t,0}\right] {W_0}^{-1}\beta_{t,0} = 0 \tag{78}$$

Since $\beta_{t,0} \neq 0$, it follows that $[{W_0}^{-1}\beta]_d \neq 0$ for some $d \in \{1, ..., D\}$, we have must have,

$$\frac{p_{l(t),1,0} - s_{t,1,0}}{[{W_0}^{-1}\beta_{t,0}]_1} = \frac{2(\beta'_{t,0}p_{l(t),0} + \gamma_{t,0})}{\beta'_{t,0}W_0^{-1}\beta_{t,0}} \tag{79}$$

We can plug (79) into (77) to obtain,

$$s_{t,0} = p_{l(t),0} - 2\left[\frac{\gamma_{t,0} + \beta'_{t,0}p_{l(t),0}}{\beta'_{t,0}W_0^{-1}\beta_{t,0}}\right] {W_0}^{-1}\beta_{t,0} \tag{80}$$

proving the first part of the result. We can multiply both sides of (71) by $\beta'_{t,0}W_0^{-1}$ to obtain,

$$\beta'_{t,0}W_0^{-1}\beta_{t,0} = \frac{-2\beta'_{t,0}(p_{l(t),0} - s_{t,0})}{\sigma_{t,0}} \tag{81}$$

[11] Here $[W_0^{-1}\beta_{t,0}]_k$ denote the k^{th} component of the vector $W_0^{-1}\beta_{t,0}$.

which we can rearrange to obtain (69). □

References

Aleman, Eduardo, Ernesto Calvo, Mark P. Jones and Noah Kaplan. 2009. "Comparing Cosponsorhsip and Roll-Call Ideal Points." *Legislative Studies Quarterly* 34:87–116.

Clinton, Joshua, Simon Jackman and Douglas Rivers. 2004. "The Statistical Analysis of Roll Call Data." *American Political Science Review* 98:355–370.

Crisp, Brian, Scott Desposato and Kristin Kanthak. 2007. "Ideal Points, Agenda Setting, and Dimensionality." Working Paper.

Poole, Keith T. 2005. *Spatial Models of Parliamentary Voting.* New York: Cambridge University Press.

Poole, Keith T. and Howard Rosenthal. 1997. *Congress: A Political Economic History of Roll Call Voting.* New York: Oxford University Press.

Rivers, Douglas. 2003. "Identification of Multidimensional Spatial Voting Models." Working Paper.

Woon, Jonathan. 2008. "Bill Sponsorship in Congress: The Moderating Effect of Agenda Positions on Legislative Proposals." *Journal of Politics* 70:201–216.