Online Appendix to "Optimal Supermajority Requirements in a Two-Party System"

A.1 - The Relationship between *ASE* and Long Run Average Policy Loss in the Dynamic Framework

In this appendix, I show that $ASE(M)$ can be interpreted as the long run average of the policy loss in the dynamic framework. Here, I will alter the notation used to accommodate the dynamic structure of policymaking. Let $\alpha = (\alpha_1, \alpha_2, ..., \alpha_N)$ denote the vector of legislator ideal points. I will denote the dependence of the policy outcomes $x(s,\alpha;M)$ and the current policy loss $\Delta(s,\alpha;M) = (x(s,\alpha;M) - \alpha_m)^2$ on the vector of ideal points α, since α may evolve over time.

To complete the dynamic structure of the model, I must specify the link between policy outcomes and status quo points. I assume that $s' = x(s,\alpha;M) + \varepsilon$ where s denotes the current status quo, s' denotes the next periods status quo, and ε is a random shock drawn from the density f_ε. I assume that the ideal points follow a stationary and ergodic process, with stationary distribution $f_\alpha(\alpha)$. I measure policy loss using the long run average of $\Delta(s,\alpha;M)$. Let f_s denote the stationary distribution of status quo points, which is defined by,

$$f_s(s';M) = \int_{(s,\alpha)} \left[f_\varepsilon(s' - x(s,\alpha;M)) f_\alpha(\alpha) f_s(s;M) \right] ds d\alpha$$

The long run average policy loss is defined by,

$$L(M) = \int_{(s,\alpha)} \Delta(s,\alpha;M) f_s(s;M) f_\alpha(\alpha) ds d\alpha$$

In the dynamic framework, f_s is exceedingly difficult to characterize analytically, even when f_ε and f_α have simple functional forms. The reason is that characterizing the stationary distribution requires solving the nonlinear functional equation specified above, which involves an integral of a piecewise function.[1]

Fortunately, $L(M)$ can easily be characterized using simulation methods. Let α^t denote the vector of ideal points in period t, s^t denote the status quo in period t, x^t denote the policy outcome in period t, and ε^t denote iid random draws from f_ε. Conditional on the observed series α^t, the model can be used to simulate a sequence of draws for s^t as follows,

[1] The equation is a functional equation because the solution is a density, f_s, which is an infinite dimensional quantity.

(1) Compute $x^t = x(s^t, \alpha^t; M)$

(2) Draw ε^t from f_ε

(3) $s^{t+1} = x^t + \varepsilon^t$

The policy loss can then be approximated using,

$$\hat{L}(M) = \tfrac{1}{T}\sum_{t=1}^{T}\Delta(s^t, \alpha^t; M)$$

This is equivalent to the expression for $ASE(M)$. Since $\Delta(s^t, \alpha^t; M)$ is a stationary and ergodic sequence, the sum converges to its expectation, which is simply $L(M)$.[2] Hence, $ASE(M)$ is an approximation to the long run average policy loss.

A.2 – Optimal Supermajority Requirements with a Forward Looking Agenda Setter

In this appendix, I show that the main result of the paper extends to the case where the agenda setter is forward looking. Specifically, I consider a two-period model, and I show that the first period policy loss is minimized using a substantial supermajority requirement.

Considering a forward looking agenda setter significantly complicates the model for two reasons. First, forward looking behavior by the agenda setter requires that the agenda setter form beliefs about the likely pivotal legislators in future sessions of the legislature. Since I am varying the supermajority requirement (and any legislator will be pivotal for some supermajority requirement), I must allow the agenda setter to form beliefs about the entire distribution of ideal points in the legislature. To make this step as simple as possible, I deviate slightly from the framework used in the paper and I consider a legislature with a continuum of legislators. The second complication is that analytical results are no longer tractable, even if simple parametric distributions are assumed for the stochastic components of the model. Consequently, I rely on numerical computation of the equilibrium.

---

[2] Ignoring measurability issues, we can represent s^t as a function of $(\alpha^{t-1}, \varepsilon^{t-1}, \alpha^{t-2}, \varepsilon^{t-2}, \alpha^{t-3}, \varepsilon^{t-3}, \ldots)$. Moreover, $(\alpha^t, \varepsilon^t)$ is stationary and ergodic by assumption. Hence, Theorem 3.35 in White (2001) implies that s^t is stationary and ergodic. The same result further implies that $\Delta(s^t, \alpha^t; M)$ is stationary and ergodic. Hence, $\tfrac{1}{T}\sum_{t=1}^{T}\Delta(s^t, \alpha^t; M) \xrightarrow{a.s.} E[\Delta(s^t, \alpha^t; M)]$ by the Ergodic Theorem (White, 2001, pg. 44), provided that $E|\Delta(s^t, \alpha^t; M)| < \infty$.

Let α_m^t denote the median legislator's ideal point in period t. Suppose that there are a continuum of legislators in period t with ideal points uniformly distributed over $[\alpha_m^t - \frac{1}{2}, \alpha_m^t + \frac{1}{2}]$. Suppose that α_m^1 is distributed uniformly over $[-\eta, \eta]$ and that $\alpha_m^2 = \lambda\alpha_m^1 + (1-\lambda)\varepsilon$ where ε is distributed uniformly over $[-\eta, \eta]$. Suppose further that $0 < \eta < \frac{1}{2}$ and $0 < \lambda < 1$. Notice that this implies that $-\frac{1}{2} \le \alpha_m^t \le \frac{1}{2}$ and that $Corr(\alpha_m^1, \alpha_m^2) = \lambda$. We can show that under these assumptions, the lower and upper pivotal legislators are given by $\alpha_l^t = \alpha_m^t + \frac{1}{2} - q$ and $\alpha_u^t = \alpha_m^t - \frac{1}{2} + q$.

It is essential to incorporate shocks to the composition of the legislature in order to make the problem interesting.[3] Here, I consider the simplest possible specification for such shocks where the median legislator is shocked but the distance between the median and the pivotal legislators remains constant.

Assume that the agenda setter is the median member of the majority party and assume that legislators with ideal points to the left of zero are Democrats and the remaining legislators are Republicans. We have,

$$a^t = \begin{cases} \frac{1}{2}\alpha_m^t + \frac{1}{4}, & \alpha_m^t > 0 \\ \frac{1}{2}\alpha_m^t - \frac{1}{4}, & \alpha_m^t < 0 \end{cases}$$

Consider a two period model version of the Setter model where the agenda setter derives utility from both periods. Let β denote the discount factor, where $0 < \beta < 1$. All legislators are assumed to myopic in their voting behavior. The policy outcome in the last period follows the static result for the Setter model,[4]

$$x^2(\alpha_m^2, s^2) = \begin{cases} a^2, & s < 2\min\{\alpha_l^2, a^2\} - a^2 \\ 2\min\{\alpha_l^2, a^2\} - s^2, & 2\min\{\alpha_l^2, a^2\} - a^2 \le s^2 \le \min\{\alpha_l^2, a^2\} \\ s^2, & \min\{\alpha_l^2, a^2\} < s^2 < \max\{\alpha_u^2, a^2\} \\ 2\max\{\alpha_u^2, a^2\} - s^2, & \max\{\alpha_u^2, a^2\} \le s^2 \le 2\max\{\alpha_u^2, a^2\} - a^2 \\ a^2, & s^2 > 2\max\{\alpha_u^2, a^2\} - a^2 \end{cases}$$

In the first period, the agenda setter must anticipate the likely outcome of the second period as a function of his first period proposal. I assume that the first period status quo is drawn from $s^1 \sim Normal(0,1)$ and that the second period status quo evolves according to

[3] Baron (1996) shows that if the composition of the legislature is stable over time, policies will eventually converge to the median legislator's position. My goal is to study institutional design under less favorable (and more realistic) circumstances, where the median legislator represents a moving target.

[4] Here, I note that a^2, α_l^2, and α_u^2 are functions of α_m^2 and q, but I suppress this dependence in the notation for simplicity.

$s^2 \sim Normal(x^1, \delta)$.[5] The agenda setter chooses b^1 to maximize his discounted utility subject to the constraint that the proposal b^1 passes,

$$b^1(s^1) = \underset{b:|\alpha_l^1 - b| \le |s^1 - b| \text{ and } |\alpha_u^1 - b| \le |s^1 - b|}{\arg\max} \left\{ -(b - a^1)^2 - \beta \int_{(\alpha^2, s^2)} (x^2(\alpha_m^2, s^2) - a^1)^2 \, dF(\alpha_m^2, s^2 \mid \alpha_m^1, b) \right\}$$

Here, the agenda setter forms his belief for α_m^2 conditional on α_m^1 and his belief for s^2 conditional on b. We further have that $x^1(s^1) = b^1(s^1)$, or that the proposal passes and becomes the first period policy outcome. Define the policy loss for the first period using,

$$L^1(q) = \int_{(s_1, \alpha_m^1)} (x^1(s^1) - \alpha_m^1)^2 \, dF(s^1, \alpha_m^1)$$

and define the optimal supermajority requirement by,

$$q^* = \underset{\frac{1}{2} \le q \le 1}{\arg\max} \left\{ L^1(q) \right\}$$

There are four steps to numerically computing the equilibrium in the model. First, $\int_{(\alpha^2, s^2)} (x^2(\alpha_m^2, s^2) - a^1)^2 \, dF(\alpha_m^2, s^2 \mid \alpha_m^1, b)$ is approximated using a raw frequency simulator. Second, $b^1(s^1)$ is calculated by numerically maximizing the agenda setter's utility over a finite grid. Third, the $L^1(q)$ is approximated using a raw frequency simulator. Finally, q^* is computed by numerically optimizing $L^1(q)$ over a finite grid.[6] The four steps to this process indicate why analytical results are so hard to obtain.

I consider the following parameter values in the illustration- $\eta = 0.3$, $\delta = 0.05$, and $\lambda = 0.8$. The parameters η and λ are chosen to generate realistic values for the variance of the median legislator's position over time and for the likelihood of a change in party control. I vary the value of β to illustrate the dependence of the optimal supermajority requirement on the degree of forward-looking behavior by the agenda setter. The numerical results are given below in Table A.1.

The results indicate that supermajority requirements are optimal even when the agenda setter displays a substantial amount of forward looking behavior. I note that I have experimented with other parameter values and have found that the optimality of supermajority requirements continues to hold (although the exact value of q^* of course depends on the parameter values).

[5] As is the case in the paper, the results in this appendix continue to hold if $\delta = 0$.

[6] I use $R = 100$ draws in both simulation steps. I consider a grid of $\{-1.00, -0.99, -0.98, ..., 0.99, 1.00\}$ for b and a grid of $\{0.500, 0.505, 0.510, ..., 0.995, 1.000\}$ for q.

Table A.1 – Optimal Supermajority Requirements with a Forward-Looking Agenda Setter

Bi-Yearly β	*Yearly* β	q^*
0.0	0.000	0.610
0.1	0.316	0.605
0.2	0.447	0.610
0.3	0.548	0.575
0.4	0.632	0.590
0.5	0.707	0.570
0.6	0.775	0.560
0.7	0.837	0.550
0.8	0.894	0.555
0.9	0.949	0.550

What value of β is most reasonable is hard to determine. I argued in the paper that a fairly low value of β is reasonable. First, it would be very difficult for members of the majority party to explain forward looking agenda setting behavior to their constituents. Moreover, there is evidence to suggest that politicians in fact have very low discount factors (Merlo, 1997). Overall, I would suggest that a bi-yearly discount factor lower than 0.5 is most reasonable.